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Conjugate heat transfer from small isothermal heat sources embedded in a large substrate

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Abstract—This paper considers the problem of conjugate heat transfer from discrete heat sources mounted on one wall of a channel and exposed to fully-developed laminar flow. Appropriate dimensionless parameters are suggested by the governing energy conservation equations and boundary conditions, and finite-difference solutions are obtained for selected parameter ranges. Two cases are considered, that of a single heat source and that of two adjoining heat sources. Parametric calculations reveal the range of conditions for which conduction heat transfer from the source to the substrate comprises a significant fraction of the total heat transfer from the source.

INTRODUCTION

THE WORK described in this paper concerns a conjugate analysis of forced convection heat transfer from small, isothermal sources embedded in a large substrate. The system of interest involves hydrodynamically fully-developed laminar flow between parallel walls, with one or two heat sources embedded in one of the walls (Fig. 1). Heat is transferred from an isothermal source at T_h to the fluid and wall (substrate) of thermal conductivities k_f and k_s, respectively. The substrate thickness is much larger than that of the sources, and the substrate and top wall boundaries are well insulated from the surroundings.

The analysis considers the effects of flow conditions, geometry, and fluid and substrate properties, and its results should be of special interest to experimentalists concerned with convection heat transfer from embedded sources. In such experiments substrate heat losses must be subtracted from the power dissipated at each source to obtain the rate of heat transfer by convection. Since the losses are difficult to measure, it is important to know conditions for which they may be neglected or, if they are not negligible, their magnitude relative to the source power.

The results of this study are also of interest in a totally different context. In certain situations, it is desirable to enhance total heat transfer from a source, which would include transfer to an adjoining substrate, as well as to a fluid. A pertinent application involves the cooling of packaged electronic components, such as very large scale integrated circuits. In such cases, the results of this study could be useful in determining total heat transfer and in suggesting means by which this transfer could be enhanced.

Although the effects of wall conduction on forced convection heat transfer have been previously considered [1–8], none of the studies concerned the problem of discrete heat sources embedded in a substrate. The only known previous work on conjugate heat transfer from small heat sources embedded in a large substrate is reported by Zinnes [9]. In that work, the conjugate problem of natural convection from a vertical surface with discrete heat sources was examined. In contrast the present situation involves forced convection in a channel and, both qualitatively and quantitatively, is substantially different from that considered by Zinnes [9].

ANALYSIS

Governing equations

Flow in the channel is assumed to be steady, laminar, two-dimensional, and hydrodynamically fully-

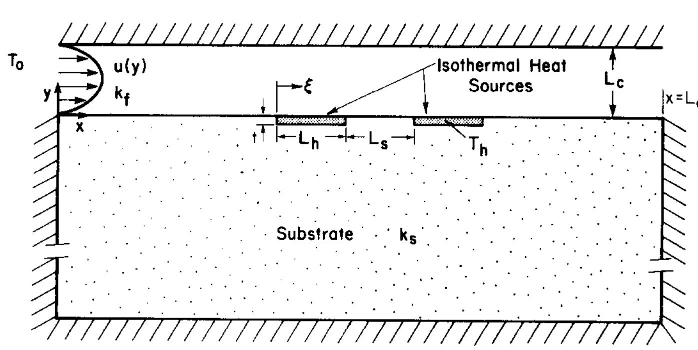

FIG. 1. Schematic of the conjugate heat transfer problem.

NOMENCLATURE

k	thermal conductivity
L_c	channel height
L_h	heat source length
L_o	length of channel
L_s	spacing between heat sources
Nu_ξ	local Nusselt number, equation (13)
$\overline{Nu}$	average Nusselt number, equation (15)
Pe	Péclet number
Q_s	rate of heat transfer to the substrate
Q_T	total rate of heat transfer from source
Q_f^*	rate of heat transfer to fluid from either of two adjoining sources normalized by rate of heat transfer to fluid from a single heat source
Q_s^*	rate of heat transfer to substrate from either of two adjoining sources normalized by rate of heat transfer to substrate from a single heat source
q	local heat flux
t	heat source thickness
T	temperature
u	fluid velocity
x, y	Cartesian coordinates (x measured from channel inlet).

Greek symbols

α	thermal diffusivity
θ	dimensionless temperature, equation (8)
ξ	streamwise coordinate with origin at leading edge of heat source.

Subscripts

a	refers to analytical solution
f	fluid
h	heat source
o	fluid inlet temperature
s	substrate.

developed. As shown in Fig. 1, the substrate thickness is much greater than that of the heat source, and its boundaries are well insulated. The channel is assumed to be very long, and its upstream and downstream boundaries are remote from the heat sources. All relevant thermophysical properties are assumed to be constant, and buoyancy effects, if any, are neglected.

The temperature fields in the fluid and substrate are governed by conservation requirements of the form

$$u\frac{\partial T_f}{\partial x} = \alpha_f\left(\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2}\right) \tag{1}$$

$$0 = \frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} \tag{2}$$

where u, the velocity distribution in fully-developed laminar flow, is given by

$$u = 6\bar{u}\left(\frac{y}{L_c} - \frac{y^2}{L_c^2}\right) \tag{3}$$

$\bar{u}$ being the mean velocity. Equations (1) and (2) are solved simultaneously, subject to the boundary conditions

$$T_f(0, y) = T_0 \tag{4}$$

$$\left.\frac{\partial T_f}{\partial x}\right|_{x=L_0} = 0 \tag{5}$$

$$\left.\frac{\partial T_s}{\partial n}\right|_{y\neq 0} = 0. \tag{6}$$

Equation (4) prescribes a uniform fluid temperature at the channel entrance, and equation (5) assumes the streamwise energy diffusion flux to be negligible at the outlet. With n representing the normal direction, equation (6) imposes an adiabatic condition at all substrate boundaries except that for which $y = 0$. The finite-difference solution procedure adopted in this work automatically satisfies the condition of heat flux continuity at the solid/fluid interface, i.e.

$$k_s\left.\frac{\partial T_s}{\partial y}\right|_{y=0} = k_f\left.\frac{\partial T_f}{\partial y}\right|_{y=0}. \tag{7}$$

A few points concerning the foregoing equations are noteworthy. The term $\partial^2 T_f/\partial x^2$ is retained in equation (1) to permit treatment of the fluid and solid regions as one domain with $u = 0$ in the solid. In addition, the retention of this term accounts for the possibility of significant streamwise conduction in the fluid in the vicinity of the heat sources. The presence of the streamwise second derivative necessitates a boundary condition at $x = L_0$. However, at moderate to large Péclet numbers, a precise specification of the temperature distribution at the downstream boundary is unnecessary. Since equation (1) is almost parabolic, the solution at upstream locations is virtually unaffected by small variations in the temperature at the outflow boundary. Equation (5) is, therefore, an adequate and realistic prescription for the outflow boundary.

Although all of the results in this paper are based on assuming negligible interfacial contact resistance between the heat source and the substrate, calculations have been performed to determine the effect of such a resistance. These calculations show that the results are virtually unaffected by the introduction of realistic contact resistances.

The governing equations may be nondimensionalized by introducing the dimensionless temperatures

$$\theta_s = \frac{T_s - T_0}{T_h - T_0} \quad \text{and} \quad \theta_f = \frac{T_f - T_0}{T_h - T_0} \tag{8}$$

and by using L_h and $\bar{u}$ as the length and velocity scales. The mean velocity $\bar{u}$ is defined by

$$\bar{u} = \frac{1}{L_c}\int_0^{L_c} u \, dy. \tag{9}$$

The governing dimensionless parameters are then of the form

$$Pe = \frac{2\bar{u}L_c}{\alpha_f}, \quad \frac{L_c}{L_h}, \quad \frac{k_s}{k_f}.$$

An additional parameter, L_s/L_h, is needed if two identical heat sources, separated by the distance L_s, are mounted on the substrate.

Solution procedure

The governing equations were solved by the control-volume finite-difference procedure described by Patankar [10]. Following this procedure, the computational domain was discretized by a set of rectangular control volumes, and a node was placed at the centroid of each. The domain encompassed both the solid substrate and the flow channel. The discretization equations were obtained by making energy balances over each control volume with the heat fluxes at the control surfaces being expressed in terms of adjacent nodal temperatures. The discontinuity in thermal conductivity at the solid/fluid interface was treated by placing a control surface at that location and utilizing the harmonic mean thermal conductivity. This procedure is described in detail in [10]. The nodes were placed either completely in the solid or completely in the fluid; none were placed at the interface. The system of algebraic discretization equations obtained for the fluid and solid regions were solved simultaneously through the line-by-line application of the tri-diagonal matrix algorithm.

Solutions were obtained for values of k_s/k_f between 0.1 and 100 and values of Pe ranging from 10^3 to 10^6. Consistent with the assumption of laminar flow, solutions for $Pe > 10^5$ correspond to high Prandtl number fluids. Most of the computations were for $L_h/L_c = 2$, although the influence of this parameter was explored separately by varying it between 0.1 and 10. The total computational domain was discretized by a non-uniform 26×100 grid for $k_s/k_f \leqslant 10$ and by a 26×190 grid for higher k_s/k_f. The values indicate, respectively, the number of horizontal and vertical grid lines. Of the 26 horizontal grid lines, 12 were deployed on the fluid side. These lines were closely spaced near the solid/fluid interface. Similarly, on the solid side the grid lines were packed near the interface, with two grid lines placed in the region corresponding to the thickness of the heat source. The vertical grid lines were also deployed non-uniformly, with 10 lines placed within the width of each heat source.

The size of the substrate was chosen to be large relative to that of the heat sources. The distance between the channel entrance and the first heat source was fixed at $7L_h$ for $k_s/k_f \leqslant 10$ and at $25L_h$ for larger values of k_s/k_f. The thickness of the substrate and the distance between the second heat source and the downstream end of the channel were fixed at $7L_h$. Numerical tests indicated that the solutions were insensitive to further increases in the dimensions of the substrate.

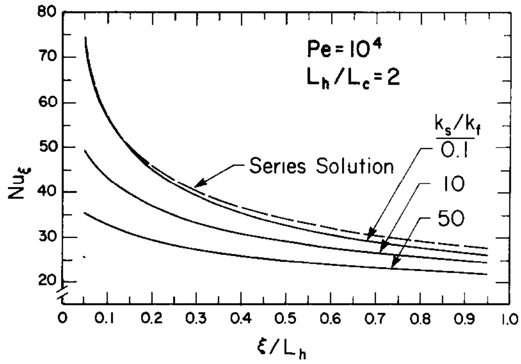

FIG. 2. Variation of local Nusselt number along the surface of the heat source for a single heat source ($Pe = 10^4$).

RESULTS

Figures 2 and 3 depict the variation of the local Nusselt number along the surface of a single heat source for $Pe = 10^4$ and 10^6, respectively. The abscissa, ξ/L_h, is zero at the upstream edge of the heat source and unity at the downstream edge. The three solid curves represent numerical results for different k_s/k_f, while the dashed line represents the analytical (infinite series) solution for $k_s/k_f = 0$. The series solutions were obtained from Shah and London [11] and are correlated by the equations

$$Nu_{\xi,a} = 1.233(\xi^*)^{-0.333} + 0.4 \quad \xi^* \leqslant 0.001 \tag{10}$$

$$Nu_{\xi,a} = 7.541 + 6.874(10^3\xi^*)^{-0.488}\exp(-245\xi^*) \quad \xi^* \geqslant 0.001 \tag{11}$$

where

$$\xi^* = \xi/(2L_c Pe). \tag{12}$$

The local Nusselt number is defined as

$$Nu_\xi = \frac{q}{T_h - T_b(\xi)} \times \frac{2L_c}{k_f} \tag{13}$$

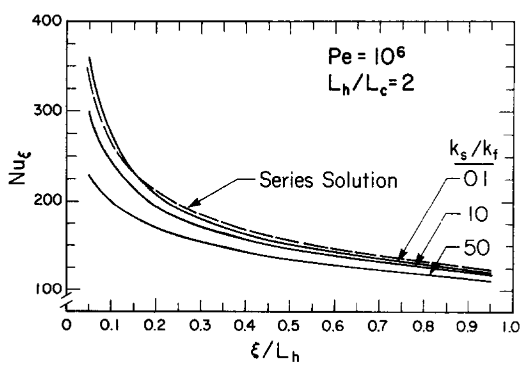

FIG. 3. Variation of local Nusselt number along the the surface of the heat source for a single heat source ($Pe = 10^6$).

where q is the local heat flux and $T_b(\xi)$ is the bulk temperature of the fluid at ξ

$$T_b(\xi) = \frac{1}{\bar{u}L_c}\int_0^{L_c} uT(\xi, y)\,dy. \tag{14}$$

From Figs. 2 and 3 it is seen that the finite-difference solution for $k_s/k_f = 0.1$ is in close agreement with the series solution for $k_s/k_f = 0$. For $Pe = 10^4$ and 10^6, respectively, differences between the two results are 5.5 and 3% at the downstream end and 3.5 and 6% at the upstream end. In view of the small Graetz numbers $(L_h/2L_cPe)$ and the corresponding steepness in longitudinal variations of the local Nusselt number, the agreement is good and confirms the accuracy of the finite-difference solution. Since substrate conduction has the effect of decreasing longitudinal variations in the Nusselt number, smaller errors are associated with increasing k_s/k_f.

Numerical results for $k_s/k_f = 10$ and 50 lie considerably below the series solution. In these cases, heat conduction in the substrate leads to the development of a thermal boundary layer upstream of the heat source. At the leading edge of the source, the thermal boundary layer is substantial, thus attenuating heat transfer from the surface of the heat source.

Figure 4 depicts the variation of the normalized average Nusselt number $\overline{Nu}/\overline{Nu}_a$ with Pe for a single heat source and several values of k_s/k_f. The Nusselt number is defined as

$$\overline{Nu} = \frac{Q}{L_h(T_h - T_0)} \times \frac{2L_c}{k_f} \tag{15}$$

where Q is the rate of heat transfer from the source to the fluid and $\overline{Nu}_a$ is the result for $k_s/k_f = 0$, evaluated from the analytical solution. For $k_s/k_f = 0.5$, the numerical and series solutions differ by less than 6% over the range of Pe. This difference increases with increasing k_s/k_f and decreasing Pe, as substrate conduction effects become more important.

In Fig. 5, the variation of Q_s/Q_T (the ratio of the rate of heat transfer by conduction into the substrate to the total rate of heat transfer from the source) with Pe is plotted for several values of k_s/k_f. The results clearly illustrate conditions for which substrate conduction can make a major contribution to the total heat transfer process. Its influence is especially pronounced at low Pe and high k_s/k_f. From the point of view of a laboratory investigator wishing to determine convection heat transfer coefficients, substrate conduction would be deleterious to the accuracy of an experiment. On the other hand, from the point of view of an electronics packaging engineer, a large conduction heat flux could be gainfully exploited in cooling electronic components mounted on a substrate.

It is appropriate to reiterate that the present computations are limited to laminar flow. In practice, laminar channel flows are influenced by buoyancy effects arising from density gradients in the fluid. Buoyancy-driven secondary flows could cause significant heat transfer augmentation at the solid/fluid interface, particularly at high heat fluxes. Such

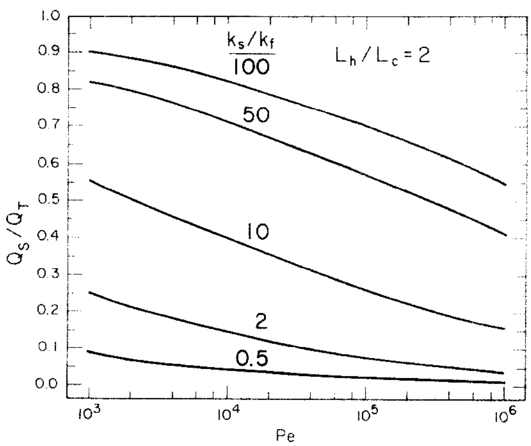

FIG. 5. Ratio of substrate heat transfer to total heat transfer vs Péclet number for different thermal conductivity ratios.

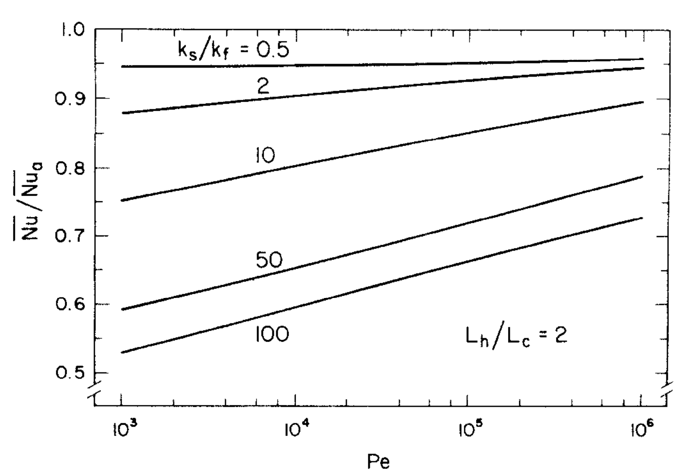

FIG. 4. Normalized averaged Nusselt number vs Péclet number for different thermal conductivity ratios.

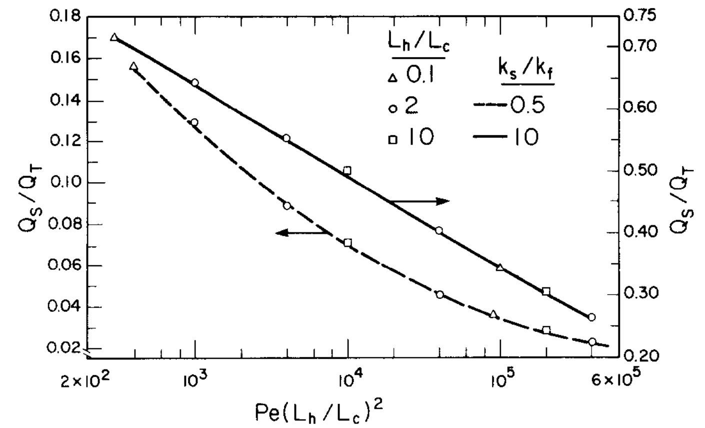

FIG. 6. Ratio of substrate heat transfer to total heat transfer vs $Pe(L_h/L_c)^2$ for various L_h/L_c and k_s/k_f.

augmentation would lead to a decrease in Q_s/Q_T below the values presented in Fig. 5. Consequently, the results of Fig. 5 represent an upper bound on the substrate conduction heat loss that may occur in practice.

The effect of the geometric parameter L_h/L_c is shown in Fig. 6, which depicts the results of calculations for different L_h/L_c and k_s/k_f. For a given value of k_s/k_f, predictions corresponding to different values of L_h/L_c and Pe collapse on a single curve when Q_s/Q_T is plotted as a function of the parameter $Pe(L_h/L_c)^2$. Accordingly, results presented in Figs. 4 and 5 for $L_h/L_c = 2$ can be applied for other values of L_h/L_c by appropriately modifying the abscissa variable of the figures.

The effect of the first heat source on heat transfer from the second source in an array of two sources is shown in Fig. 7. Results are presented in terms of variation of the normalized heat transfer rate Q_f^* with the heat source spacing parameter L_s/L_h. The quantity Q_f^* represents the rate of heat transfer from either heat source to the fluid normalized by the corresponding rate of heat transfer to the fluid for the case of a single heat source. The two halves of the figure correspond to different values of k_s/k_f. Since Q_f^* for the first heat source is equal to one, irrespective of L_s/L_h, k_s/k_f, or Pe, the second heat source clearly has a negligible effect on heat transfer from the first source. However, the presence of the first heat source has a pronounced effect on heat transfer from the second source. For small values of L_s/L_h, Q_f^* is significantly less than one. As L_s/L_h increases, Q_f^* increases and, in the limit of large spacing, would be asymptotic to unity. In contrast to the effect of L_s/L_h, the influence of Pe is small. The behavior depicted in this figure is, of course, to be expected in view of the almost parabolic nature of the governing energy equation on the fluid side.

Variation of the normalized conduction heat transfer rate Q_s^* with L_s/L_h is presented in Fig. 8. Once again, conditions for the first heat source are virtually unaffected by presence of the second source, except at very small values of L_s/L_h. However, substrate conduction heat transfer from the second heat source is strongly influenced by the presence of the first source.

The ratio Q_s/Q_T for each of the two heat sources is presented in Fig. 9 as a function of L_s/L_h. For $L_s/L_h > 2$, both heat sources have the same value of Q_s/Q_T, and the single heat source results of Fig. 5 could be applied for each source. If $L_s/L_h < 2$, thermal interaction between the two heat sources causes differences in the two values of Q_s/Q_T. However, differences remain small for values of L_s/L_h as low as 0.2.

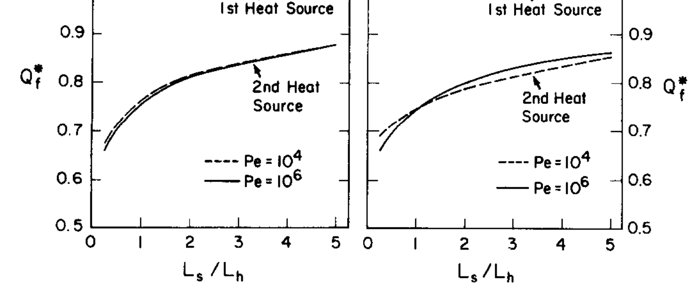

FIG. 7. Variation of normalized rate of heat transfer to the fluid with L_s/L_h for each of two adjoining heat sources.

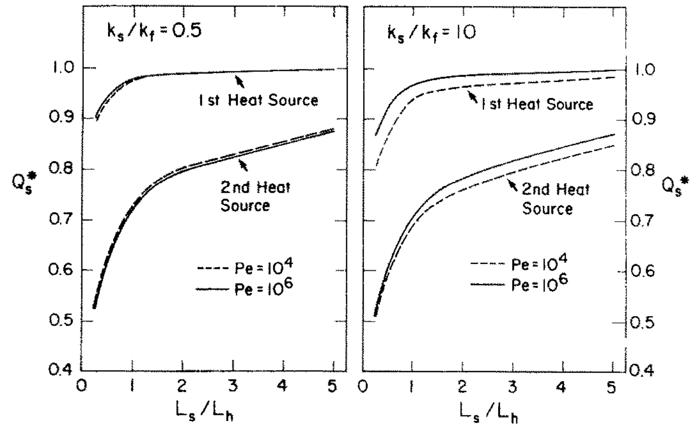

FIG. 8. Variation of normalized rate of heat transfer to the substrate with L_s/L_h for each of two adjoining heat sources.

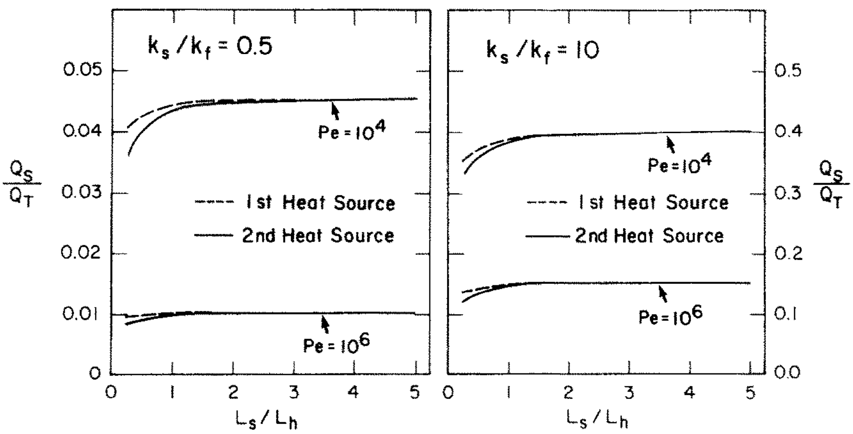

FIG. 9. Ratio of substrate to total heat rate vs L_s/L_h for each of two adjoining heat sources.

Figures 10–13 indicate variation of the local Nusselt number along the solid/fluid interface for both the single and dual heat source cases. In Fig. 10, the normalized local Nusselt number Nu/Nu_{fd} for a single heater is presented for three values of k_s/k_f. The local

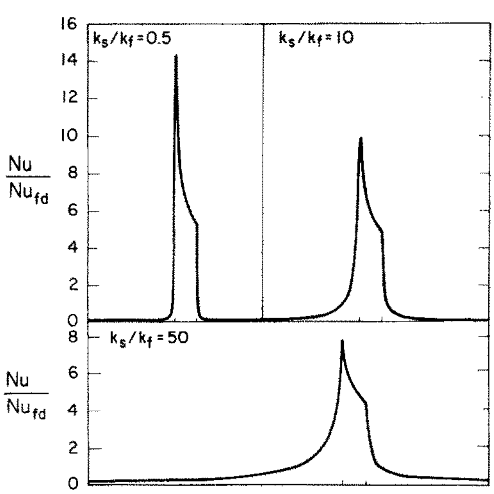

FIG. 10. Variation of normalized Nusselt number along the fluid/solid interface for a single heat source ($Pe = 10^4$).

Nusselt number Nu is defined by

$$Nu = \frac{q(x)}{(T_h - T_0)} \times \frac{2L_c}{k_f} \tag{16}$$

and Nu_{fd} is the Nusselt number for fully-developed flow and heat transfer in a channel with one wall isothermal and the other adiabatic ($Nu_{fd} = 4.86$). The tick marks on the abscissae indicate the location of the heat source.

For $k_s/k_f = 0.5$, the Nusselt number distribution is sharply peaked at the upstream edge of the heat source and drops steeply towards the downstream edge. The Nusselt number at the surface of the adjoining substrate is almost zero, indicating that heat conduction into the substrate is minimal. For $k_s/k_f = 10$, the Nusselt number decreases at the leading edge and becomes more discernible in the adjoining substrate. This result is due to substrate conduction. Clearly, any energy conducted into the substrate must reappear at the substrate/fluid interface. This behavior attenuates the nearly discontinuous variation of Nu predicted at the leading edge for $k_s/k_f = 0.5$. At $k_s/k_f = 50$ smoothing of the discontinuity in the distribution of Nu is much greater and the magnitude of the leading-edge Nusselt number is greatly diminished. In contrast, the value of

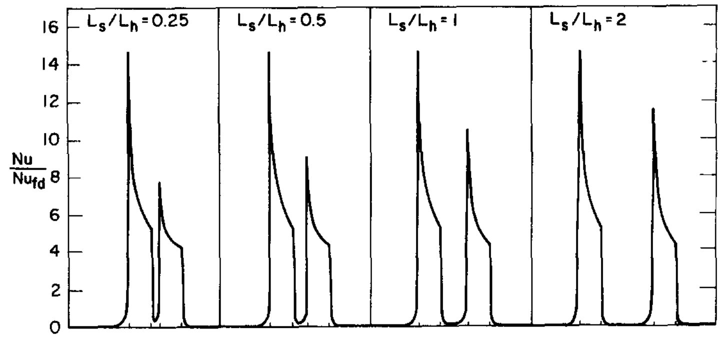

FIG. 11. Variation of normalized Nusselt number along the fluid/solid interface for two heat sources ($Pe = 10^4$, $k_s/k_f = 0.5$).

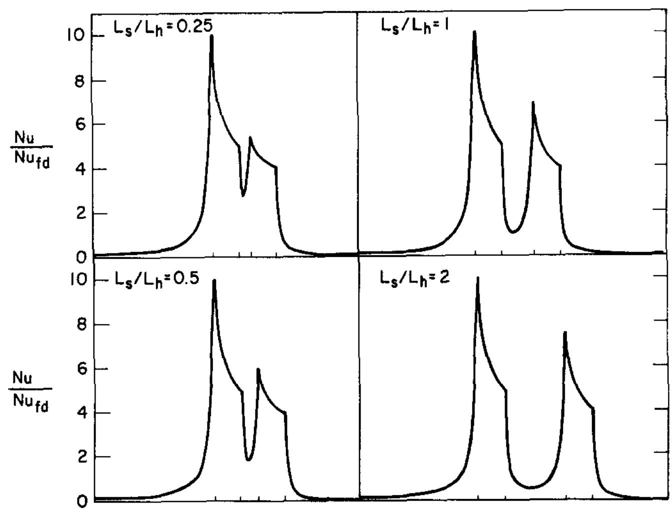

FIG. 12. Variation of normalized Nusselt number along the fluid/solid interface for two heat sources ($Pe = 10^4$, $k_s/k_f = 10$).

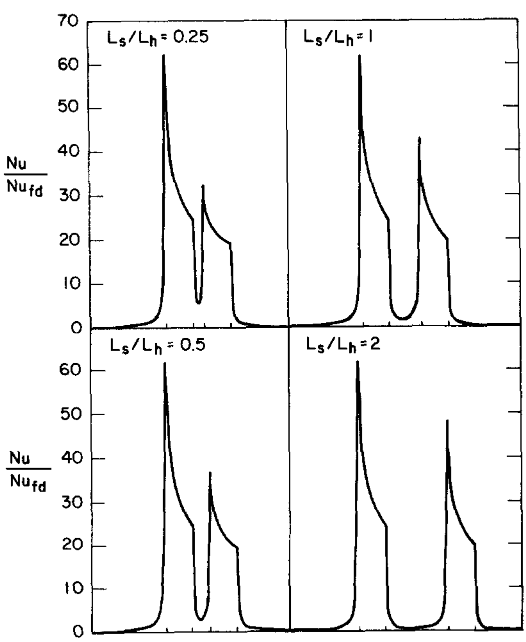

FIG. 13. Variation of normalized Nusselt number along the fluid/solid interface for two heat sources ($Pe = 10^6$, $k_s/k_f = 10$).

Nu at the trailing edge of the heat source is only weakly affected by the value of k_s/k_f.

Figures 11–13 present local Nusselt number distributions for the two heat source case. In Fig. 11 ($Pe = 10^4$ and $k_s/k_f = 0.5$), it is observed that for small values of L_s/L_h the Nusselt number at the leading edge of the second source is greatly reduced due to preheating of the fluid by the first heat source. With increasing L_s/L_h this Nusselt number recovers and approaches that of the first peak. It is also observed that substrate conduction plays a progressively smaller role and the Nu distributions become more sharply peaked.

For comparison, Nusselt number distributions for $Pe = 10^4$ and $k_s/k_f = 10$ are presented in Fig. 12. As for $k_s/k_f = 0.5$, the Nusselt number at the leading edge of the second source is greatly reduced for small values of L_s/L_h but recovers as L_s/L_h is increased. Also, the Nusselt number at the leading edge of the first source is unaffected by the presence of the second source. However, significant substrate conduction effects are observed. There is substantial heat transfer from the solid substrate adjoining both heat sources, and substrate conduction is especially important for small values of L_s/L_h.

The effect of Pe is shown in Fig. 13. Comparing the results with those of Fig. 12, the principal differences are increasing Nu and decreasing substrate conduction with increasing Pe. The latter effect is clearly revealed by the Nu profiles along the substrate between the two heat sources.

CONCLUSION

The problem of conjugate heat transfer from discrete heat sources mounted on a solid substrate and exposed to a flowing fluid has been investigated numerically. The results indicate that, for high values of the substrate-to-fluid thermal conductivity ratio and for low fluid Péclet numbers, conduction heat transfer from the source to the substrate can make a major contribution to the total heat transfer from the source. If two heat sources are mounted side by side, the heat transfer from the first source is unaffected by the presence of the second source, for the ranges of the parameters explored in this study. The second heat source is, however, strongly influenced by the first source. The results of this paper should be useful to laboratory studies requiring knowledge of the extent of conduction heat loss in forced convection experiments. The results are also useful to electronic packaging engineers wishing to enhance the cooling of electronic components mounted on a solid substrate.

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TRANSFERT THERMIQUE CONJUGUE A PARTIR DE PETITES SOURCES ISOTHERMES DE CHALEUR NOYEES DANS UN GRAND SUBSTRAT

Résumé—On considère le problème de transfert thermique conjugué à partir de sources de chaleur discrètes montées sur une paroi d'un canal exposée à un écoulement laminaire complètement établi. Des paramètres adimensionnels appropriés sont suggérés par les équations de conservation d'énergie et les conditions aux limites; des solutions par différences finies sont obtenues pour les domaines de paramètres choisis. On considère deux cas, celui d'une source unique de chaleur et celui de deux sources adjointes. Des calculs paramètriques révèlent le domaine des conditions pour lequel la conduction thermique depuis la source dans le substrat représente une fraction sensible du transfert thermique partant de la source.

WÄRMEÜBERGANG VON KLEINEN ISOTHERMEN WÄRMEQUELLEN AUF EINER AUSGEDEHNTEN UNTERLAGE

Zusammenfassung—Die Arbeit behandelt das Problem des Wärmeüberganges von diskreten Wärmequellen an einer Wand eines Kanals bei vollständig ausgebildeter laminarer Strömung. Aus der Energiebilanzgleichung und den Randbedingungen werden geeignete dimensionslose Parameter abgeleitet. Für ausgewählte Parameterbereiche ergeben sich Lösungen auf der Basis von finiten Differenzen. Zwei Fälle werden betrachtet, derjenige einer Einzelwärmequelle und derjenige von zwei benachbarten Wärmequellen. Parameterrechnungen zeigen den Bereich von Bedingungen auf, in dem der Wärmetransport durch Leitung einen signifikanten Anteil des Gesamtwärmetransports darstellt.

СОПРЯЖЕННЫЙ ТЕПЛОПЕРЕНОС ОТ НЕБОЛЬШИХ ИЗОТЕРМИЧЕСКИХ ИСТОЧНИКОВ ТЕПЛА, РАЗМЕЩЕННЫХ ВБЛИЗИ ПОВЕРХНОСТИ БОЛЬШОГО СЛОЯ

Аннотация—В работе рассматривается задача сопряженного теплопереноса от дискретного источника тепла, установленного на одной стенке канала и омываемого полностью развитым ламинарным потоком. Предложены соответствующие безразмерные параметры и граничные условия, а решения в конечно-разностном виде получены для выбранной области параметров. Рассматривается два случая: с единичным источником тепла и с двумя примыкающими тепловыми источниками. В результате параметрических расчетов найден диапазон условий, для которого кондуктивный теплоперенос от источника к подложке составляет существенную часть суммарного теплообмена от источника.